Multiplying and Dividing Radicals

Objective To multiply and divide radical expressions

Lesson Vocabulary
- simplest form of a radical
- rationalize the denominator

If the radicand of $\sqrt[n]{a}$ has a perfect nth power among its factors, you can *reduce* the radical. If you reduce a radical as much as possible, the radical is in **simplest form**. For example, consider $\sqrt{24}$ and $\sqrt[3]{24}$.

$$\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{2^2 \cdot 6} = \sqrt{2^2} \cdot \sqrt{6} = 2\sqrt{6}$$ $2\sqrt{6}$ is in simplest form.

$$\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} = \sqrt[3]{2^3 \cdot 3} = \sqrt[3]{2^3} \cdot \sqrt[3]{3} = 2\sqrt[3]{3}$$ $2\sqrt[3]{3}$ is in simplest form.

Another way to simplify a radical expression is to **rationalize the denominator**. You rewrite the expression so that there are no radicals in any denominator and no denominator in any radical.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$$

Essential Understanding:

You can simplify radicals that have the same index

Roots & Radicals

$\sqrt{x}$

Number	Squared	Square Root
1	1	1
2	4	2
3	9	3
4	16	4
5	25	5
6	36	6
7	49	7
8	64	8
9	81	9
10	100	10
11	121	11
12	144	12
13	169	13

$\sqrt[3]{x}$

Number	Cube	Cube Root
1	1	1
2	8	2
3	27	3
4	64	4
5	125	5
6	216	6
7	343	7
8	512	8
9	729	9
10	1000	10
11	1331	11
12	1728	12
13	2197	13

Prime Factorization

12

16

48

90

Multiplying and Dividing Radicals

take note

Property **Combining Radical Expressions: Products**

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

Same Index

$\sqrt{2} \cdot \sqrt{3} = \sqrt{2 \cdot 3}$

$\sqrt[3]{4} \cdot \sqrt[3]{5} = \sqrt[3]{4 \cdot 5}$

Multiplying and Dividing Radicals

Can you simplify the product of the radical expressions? Explain.

A $\sqrt[3]{6} \cdot \sqrt{2}$

B $\sqrt[3]{-4} \cdot \sqrt[3]{2}$

No. The indexes are different.

Yes. $\sqrt[3]{-4} \cdot \sqrt[3]{2} = \sqrt[3]{-4(2)} = \sqrt[3]{-8} = -2.$

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Problem 3 **Simplifying a Product**

What is the simplest form of $\sqrt{72x^3y^2} \cdot \sqrt{10xy^3}$?

$$\begin{aligned}\sqrt{72x^3y^2} \cdot \sqrt{10xy^3} &= \sqrt{(72x^3y^2)(10xy^3)} \\ &= \sqrt{720x^4y^5} \\ &= \sqrt{12^2(5)(x^2)^2(y^2)^2y} \\ &= \sqrt{12^2(x^2)^2(y^2)^2} \cdot \sqrt{5y} \\ &= 12|x^2y^2| \cdot \sqrt{5y} \\ &= 12x^2y^2\sqrt{5y}\end{aligned}$$

The simplest form is $12x^2y^2\sqrt{5y}$.

Multiplying and Dividing Radicals

Got It? 3. What is the simplest form of $\sqrt{45x^5y^3} \cdot \sqrt{35xy^4}$?

Multiplying and Dividing Radicals

Essential Understanding:

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take note

Property **Combining Radical Expressions: Quotients**

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, then $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$.

Multiplying	Dividing
$\sqrt{2} \cdot \sqrt{3} = \sqrt{2 \cdot 3}$	$\frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$
$\sqrt[3]{4} \cdot \sqrt[3]{5} = \sqrt[3]{4 \cdot 5}$	$\frac{\sqrt[3]{4}}{\sqrt[3]{5}} = \sqrt[3]{\frac{4}{5}}$

Multiplying and Dividing Radicals

What is the simplest form of the quotient?

A $\dfrac{\sqrt{18x^5}}{\sqrt{2x^3}}$

B $\dfrac{\sqrt[3]{162y^5}}{\sqrt[3]{3y^2}}$

$$\frac{\sqrt{18x^5}}{\sqrt{2x^3}} = \sqrt{\frac{18x^5}{2x^3}}$$
$$= \sqrt{9x^2}$$
$$= 3x$$

$$\frac{\sqrt[3]{162y^5}}{\sqrt[3]{3y^2}} = \sqrt[3]{\frac{162y^5}{3y^2}}$$
$$= \sqrt[3]{54y^3}$$
$$= \sqrt[3]{27y^3} \cdot \sqrt[3]{2}$$
$$= \sqrt[3]{3^3y^3} \cdot \sqrt[3]{2}$$
$$= 3y\sqrt[3]{2}$$

Multiplying and Dividing Radicals

Got It? 4. a. What is the simplest form of $\frac{\sqrt{50x^6}}{\sqrt{2x^4}}$?

Multiplying and Dividing Radicals

Another way to simplify a radical expression is to **rationalize the denominator**. You rewrite the expression so that there are no radicals in any denominator and no denominator in any radical.

Multiply by 1.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$$

The product of $\sqrt{2}$ and itself is a rational number, 2.

Rationalizing the Denominator

$$\frac{3}{\sqrt[5]{x^2}} \cdot \frac{\sqrt[5]{x^3}}{\sqrt[5]{x^3}} =$$

$$\frac{7}{\sqrt[6]{x^5}} \cdot \frac{\sqrt[6]{x}}{\sqrt[6]{x}} =$$

$$\frac{8}{\sqrt[8]{x^4}} \cdot \frac{\sqrt[8]{x^4}}{\sqrt[8]{x^4}} =$$

Rationalizing the Denominator

$$\frac{x}{\sqrt[2]{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} =$$

$$\frac{x}{\sqrt[3]{9}}$$

$$\frac{y}{\sqrt[4]{8}}$$

Multiplying and Dividing Radicals

Problem 5 Rationalizing the Denominator

Multiple Choice What is the simplest form of $\sqrt[3]{\frac{5x^2}{12y^2z}}$?

Think

How do you choose what to multiply by?
Choose a cube root with a radicand that will make each factor of the radicand in the denominator a perfect cube.

Multiplying and Dividing Radicals

Got It? **5. a.** What is the simplest form of $\dfrac{\sqrt[3]{7x}}{\sqrt[3]{5y^2}}$?

take note

Properties — Properties of Exponents

- $a^0 = 1, a \neq 0$
- $\frac{a^m}{a^n} = a^{m-n}$
- $a^{-n} = \frac{1}{a^n}$
- $(ab)^n = a^n b^n$
- $(a^m)^n = a^{mn}$
- $a^m \cdot a^n = a^{m+n}$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Simplify each expression. Use only positive exponents.

1. $(2a^3)(5a^4)$

2. $(-3x^2)(-4x^{-2})$

3. $(3x^2y^3)^2$

4. $(3x^{-4}y^3)^2$

5. $\frac{4a^8}{2a^4}$

6. $\frac{12x^5y^3}{4x^{-1}}$

7. $\frac{(6x^3)^0}{3xy^2}$

8. $\left(\frac{2x^4}{3}\right)^3$

9. $(-4m^2n^3)(2mn)$

10. $(2x^3y^7)^{-2}$

11. $\frac{(3r^{-2}s^3t^0)^{-3}}{3rs}$

12. $(h^7k^3)^0$

13. $\frac{r^2s^4t^6}{r^3s^4t^{-6}}$

14. $\frac{x^2y}{4} \cdot \frac{16x}{y}$

15. $(s^4t)^2(st)$

16. $\left(\frac{1}{h^{-2}}\right)^{-1} \cdot h^3$

17. $\frac{1}{a^2b^{-3}}(a^2b^{-3})^{-1}$

18. $\left(\frac{r^{-1}s^2t^{-3}}{r^{-2}s^0t^1}\right)^{-1}$

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ANY QUESTIONS?

Multiplying and Dividing Radicals

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Classwork:

Workheet 11.2

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